A characterization of possibility domains in strategic voting*

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Abstract

We consider domains that satisfy three properties, namely top-connectedness, pervasiveness, and richness. We prove the following two results for such a domain: (i) it admits non-dictatorial, unanimous, and strategy-proof choice functions if and only if it has an inseparable top-pair, and (ii) it admits anonymous, unanimous, and strategy-proof choice functions only if it does not have any top-circuit. We also provide some applications of our results on Euclidean domains, domains on convex polytopes, and domains that arise in the context of preference aggregation problems.

Keywords: top-connectedness, dictatorial domains, Euclidean preferences

1 Introduction

This paper deals with standard social choice problems where an alternative has to be chosen based on the preferences of the individuals in a society. Examples of such problems include electing a candidate for the parliament, deciding a policy for a country, finding a location for public goods such as hospitals, schools, bus-stops, etc., or public bads such as nuclear plants, garbage dumps, etc. A procedure that maps every collection of individual preferences to a feasible alternative is called a choice function. A choice function is called unanimous if, whenever all the individuals agree on their best alternatives, that alternative is chosen. It is called strategy-proof if no individual can bend its outcome in his/her favor by misreporting his/her preferences, and is called dictatorial if it always chooses the best alternative of a particular agent. We call a domain a possibility domain if it admits a non-dictatorial, unanimous, and strategy-proof choice function, otherwise we call it an impossibility domain.

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Gibbard (1973) and Satterthwaite (1975) show that if there are at least three alternatives, then the unrestricted domain is an impossibility domain. A domain is called unrestricted if it contains *all* strict preferences over the alternatives. In response to this impossibility result, several relaxations of the unrestricted domain assumption have been investigated in the literature. It is worth noting that whether a domain is impossibility or not depends on the presence or absence of certain combinations of preferences, and not on the number of preferences in it. For instance, if the number of alternatives m is at least three, then one can construct an impossibility domain with even six preferences (see e.g. Ozdemir and Sanver (2007) or Storcken (1985)), whereas a possibility domain can be constructed with as many as $m! - (m-1)! + (m-2)!$ preferences (see Aswal et al. (2003)).

Various sufficient conditions for possibility or impossibility domains are known in the literature. However, such conditions are often not enough transparent (see e.g. Sen et al. (1969), Kalai and Muller (1977), or Kalai and Ritz (1980)), which makes it hard to find their applications. In this paper, we impose three conditions, namely top-connectedness, pervasiveness, and richness, on the domains and present a complete characterization of all possibility domains. In what follows, we present a detailed description of our results. Let us use the following notation to ease our presentation. For a preference P and two alternatives a and b, we write $P \equiv ab\cdots$ to mean that the best and the second-ranked alternatives in P are a and b, respectively.

The notion of top-connectedness is introduced in Aswal et al. (2003). Two alternatives a and b are called top-connected in a domain if there are two preferences P and P' in that domain such that $P \equiv ab\cdots$ and $P' \equiv ba\cdots$. Note that P and P' need not have the same ordering over the alternatives other than a and b. Top-connectedness induces a graph structure over the alternatives in a natural way: two alternatives form an edge if and only if they are top-connected. We refer to such a graph as the top-graph of the corresponding domain. Note that the top-graph of the unrestricted domain is the complete graph and that of the (maximal) single-peaked domain is a line graph. A domain is called top-connected if its top-graph is connected, that is, any two alternatives in its top-graph are connected by a sequence of edges. A domain is said to be pervasive if, whenever there is a preference $P \equiv ab\cdots$ in that domain, there is another preference $P' \equiv ba\cdots$ in it. In other words, if such a domain allows (by admitting a preference) a to be ranked first and b to be ranked second, then it also allows b to be ranked first and a to be ranked second. Our richness condition is somewhat involved. Roughly speaking, it ensures that certain type of preferences are present in the domain. This condition is satisfied, for instance, if the domain is strongly top-connected (see Remark 2.1) or locally connected. The difference between top-connected and strongly top-connected domains is that in case of the latter, two alternatives a and b share an edge if and only if there are preferences $P \equiv ab\cdots$ and $P' \equiv ba\cdots$ in the domain satisfying the additional requirement that they agree on the ranking of the alternatives other than a and b. A domain is locally connected if for every two preferences in it, there is a sequence of preferences starting from the former and ending at the latter such that every two consecutive preferences in that sequence differ by the ordering of exactly two (consecutively ranked) alternatives.

It is worth mentioning that the conditions discussed above can be considered as mild ones in the sense that almost all domains of practical importance such as single-peaked (Moulin (1980)), single-dipped (Peremans and Storcken (1999)), single-crossing (Saporiti (2014)), circular (Sato (2010)), etc., satisfy these conditions.

Next, we discuss the notion of inseparable top-pairs which we use in our characterization result. This notion is introduced in Kalai and Ritz (1980) in the context of welfare functions. A pair of alternatives (a, b) is called an inseparable top-pair in a domain if, whenever a appears as the best alternative in a preference, b appears as the second-ranked one in it (see also Aswal et al. (2003) and Bochet and Storcken (2008)). Note that if (a, b) is an inseparable top-pair in a domain, then a will have exactly one edge in the top-graph of that domain. In such situations, we say that the top-graph has a "loose end" at a.

Theorem 1 of this paper shows that a domain satisfying top-connectedness, pervasiveness, and richness is a possibility domain if and only if it has an inseparable top-pair. Aswal et al. (2003) introduce the concept of linked domains and show that these domains are impossibility domains. A domain is linked if the alternatives can be arranged in a sequence so that in the top-graph of that domain (i) first two alternatives share an edge, and (ii) every alternative from the third position shares edges with at least two alternatives that appear before that alternative in that sequence. Clearly, such a domain does not have an inseparable top-pair. However, their result cannot be derived as a corollary of Theorem 1 since they do not assume any additional restriction on the domains. On the other hand, linked domains require a lot more structure on the top-graph as compared to the requirement that there are no loose ends in it. Thus, our result too does not follow from their result. In other related works, Sato (2010) shows that circular domains are impossibility domains, and Pramanik (2015) shows that β and γ domains are impossibility. It is worth mentioning that all these results assume the domains to be regular, that is, for every alternative they have a preference that places it at the top position. However, our results apply to the domains that are not necessarily regular. In Section 5, we provide a formal discussion on the connection of our result with these closely related papers.

Note that possibility domains do not guarantee choice functions that have other desirable properties such as anonymity. In fact, there are possibility domains that admit unanimous and strategy-proof choice functions for which a particular agent behaves like a dictator for all but one profiles (see Example 3.1). This motivates us to provide some structure of the domains that admit choice functions that are anonymous as well as unanimous and strategy-proof. Theorem 2 of this paper provides a necessary condition for this. It says that a domain admits such a choice function only if its top-graph does not contain a cycle. In case of Euclidean preferences, the restriction that there is no cycle in the top-graph implies that the domain is a set of single-peaked preferences on a tree as defined in Demange (1982). Thus, for such domains, Theorem 2 provides a necessary and sufficient condition for admitting an anonymous, unanimous, and strategy-proof choice function (see Corollary 2).

Finally, we provide three types of scenarios where our results can be applied. The first type deals with policy making problems. A policy can be identified with a point in a finite dimensional Euclidean space. In contrast to the usual setting where the set of (available) policies is infinite, here we take it to be finite. The second type concerns the problem of locating a public facility in a Euclidean space from which agents derive negative externalities at individual levels. Examples of such facilities include garbage dump, nuclear plant, wind mill, etc. Since agents derive negative externalities from such facilities, they want them to be located as far as possible from their own locations/residences. The third type of scenario arises in the context of preference aggregation problem where a list/ranking of some candidates or contestants has to be prepared based on the preferences or judgments of the agents.

The organization of this paper is as follows. Section 2 presents the basic model and the conditions that we impose on the domains. Section 3 presents the main results and Section 4 discusses some applications of those. Section 5 concludes the paper with a discussion on the connection of our results with related results. All the proofs are relegated to the appendices.

2 Preliminaries

Let N denote a non-empty set of *agents* and let A denote a non-empty set of *alternatives*. We assume that N is finite, whereas A may be finite or infinite. A preference is a linear order[1] on A. Note that only strict preferences are considered in this paper. We denote by $\mathbb{L}$ the set of all linear orders or preferences. All agents have the same domain $\mathbb{D}$ of individual admissible preferences. A profile $p \in \mathbb{D}^N$ is an N-tuple of individual preferences in $\mathbb{D}^N$. A *collective choice function* or *choice function* φ is a mapping from $\mathbb{D}^N$ to A assigning to every profile p an alternative $\varphi(p)$ in A.

The following notions and notations are used throughout the paper. For a preference R and two alternatives a and b in A, we write $ab \in R$ (instead of $(a, b) \in R$) to mean that a is (weakly) preferred to b at R. Also, we write $R \equiv \cdots ab \cdots$ to mean a is ranked *just* above b in R. Note that when we write $ab \in R$, we do not require a and b to be distinct, however when we write $R \equiv \cdots ab \cdots$, we do mean that a and b are distinct. In a similar fashion, we write $R \equiv ab \cdots$ to mean a and b are the best and the second-best alternatives, respectively, in R.[2] Notations like $R \equiv \cdots a \cdots b \cdots$, $R \equiv a \cdots$, $R \equiv \cdots a$, etc., have self

[1] A linear order is a reflexive, transitive, antisymmetric, and complete relation on A.
[2] An alternative a is said to be the best alternative at a preference R if $ab \in R$ for every alternative b in A.

explanatory interpretations.

For an alternative a, the set of linear orders in a domain $\mathbb{D}$ having a as the best alternative is defined as $\mathbb{D}^a = \{R \in \mathbb{D} : R \equiv a \cdots\}$. We define $\tau(\mathbb{D}) = \{a \in A : \mathbb{D}^a \neq \emptyset\}$ as the set of alternatives that appear as the best alternative at some preference in $\mathbb{D}$. We assume throughout this paper that $\tau(\mathbb{D})$ is a finite set. The *(weak) upper contour set of an alternative a at* a preference R is defined as $U(a, R) = \{b \in A : ba \in R\}$.

For a profile p and an agent i, we denote by $p(i)$ the preference of i in p. Suppose that S is a subset of N, and R^1 and R^2 are two preferences in $\mathbb{D}$. Then, the N-tuple $((R^1)^S, (R^2)^{N\setminus S})$ denotes the profile, say r, such that $r(i) = R^1$ for all agents i in S and $r(i) = R^2$ for all agents i in $N \setminus S$. We call such a profile an $(S, N \setminus S)$-*unanimous* profile. Additionally, if $R^1 \in \mathbb{D}^a$ and $R^2 \in \mathbb{D}^b$, then we call it an ab-$(S, N \setminus S)$-*unanimous* profile. Such a profile is called unanimous if S is empty or equal to N. The restriction of a profile p to S is denoted by $p|_S$. For a profile p, notations like $((R^1)^S, p|_{N\setminus S})$ have obvious interpretations and will be referred as an S-*unanimous* profile .

2.1 Top-graph, inseparable top-pair, and top-circuit

In this sub section, we introduce the notion of the top-graph of a domain and discuss a few properties of a domain that are based on this notion. The top-graph of a domain is defined using the notion of top-connectedness introduced in Aswal et al. (2003). Two distinct alternatives a and b in $\tau(\mathbb{D})$ are called top-connected in a domain $\mathbb{D}$ if there are two preferences R and R' in $\mathbb{D}$ such that $R \equiv ab \cdots$ and $R' \equiv ba \cdots$. We use the notation $a \leftrightsquigarrow b$ to mean that a and b are top-connected.

The top-graph of a domain $\mathbb{D}$ is defined as the graph with the set of vertices as $\tau(\mathbb{D})$ such that there is an edge between two alternatives in $\tau(\mathbb{D})$ if and only if they are top-connected. A sequence $c^0, c^1, \ldots, c^k$ of alternatives in $\tau(\mathbb{D})$ is called a *top-connecting path* (*from c^0 to c^k*) if $c^{t-1} \leftrightsquigarrow c^t$ for all $1 \leq t \leq k$. A top-connecting path $c^0, c^1, \ldots, c^k$ is called a *top-circuit* if $k \geq 3$, $c^0 = c^k$, and $c^s \neq c^t$ for all $1 \leq s < t \leq k$. A domain $\mathbb{D}$ is called *top-connected* if for all $a, b \in \tau(\mathbb{D})$, there is a *top-connecting path* from a to b.

Kalai and Ritz (1980) introduce the notion of inseparable pairs in the context of welfare functions. A pair ab of distinct alternatives in $\tau(\mathbb{D})$ is called an *inseparable top-pair in* $\tau(\mathbb{D})$ if $bc \in R$ for all $R \in \mathbb{D}^a$ and all $c \in \tau(\mathbb{D}) \setminus \{a\}$. That is, at every preference where a is the best alternative, b is preferred to every other alternative in $\tau(\mathbb{D})$.

2.2 Conditions on domains

We impose some conditions on the domains that we consider in this paper. We formulate these as Condition 1.

A domain is said to satisfy *pervasiveness* if for all distinct alternatives a and b in $\tau(\mathbb{D})$, if there is a preference $R \equiv ab\cdots$ in the domain, then there is another preference $R' \equiv ba\cdots$ in it.

A domain is said to satisfy *richness* for all distinct alternatives a, b, and c in $\tau(\mathbb{D})$ with $a \leftrightsquigarrow b$ and $b \leftrightsquigarrow c$, there are R^a in $\mathbb{D}^a$ and R^c in $\mathbb{D}^c$ such that for all $d \in U(c, R^a) \cap U(a, R^c)$, there is a preference R^b in $\mathbb{D}^b$ with the property that $d \in U(a, R^b) \cup U(c, R^b)$.

For an illustration of this condition, consider a triplet of alternatives (a, b, c) such that a and b are top-connected and b and c are top-connected. Richness says that for such a triplet, there must be two preferences R^a and R^c in the domain with the best alternatives as a and c, respectively, such that for each alternative d that is preferred to both c in R^a and a in R^c, there is a preference R^b in the domain having the best alternative as b with the property that d is preferred to at least one of a and c. For an example, suppose that there are four alternatives a, b, c, and d, and consider the domain $\{abdc, bacd, bcda, cbda, cdba, dcba\}$. We show that the domain is rich. The pairs of alternatives that are top-connected in this domain are (a, b), (b, c), and (c, d). So, we have to check the requirement of the richness property for the triplets (a, b, c) and (b, c, d). Consider the triplet (a, b, c). Take $R^a = abdc$ and $R^c = cbda$. Note that b and d are the only alternatives that are preferred to both c in R^a and a in R^c. For each of these alternatives, richness requires us to find a preference R^b with b as the best alternative where that alternative is preferred to either a or c. For b, clearly each of the preferences $bacd$ and $bcda$ does this job, and for d, the preference $bcda$ does it. Similarly, one can check that the richness property is satisfied over the remaining triplet (b, c, d) and conclude that the domain presented here is rich.

The following remark says that the richness property is automatically satisfied if we strengthen the notion of top-connectedness.

REMARK 2.1. We say two distinct alternatives a and b are strongly top-connected if there are preferences $R^a \equiv ab\cdots$ and $R^b \equiv ba\cdots$ such that for all $c, d \notin \{a, b\}$, $cd \in R^a$ if and only if $cd \in R^b$. So, preferences R^a and R^b differ only by the ranking of their top two alternatives, which are swapped from one to another. Define the strong top-graph of a domain as the one where two alternatives in $\tau(\mathbb{D})$ share an edge if and only if they are strongly top-connected. We say a domain is strongly top-connected if its strong top-graph is connected. With slight abuse of notation, we use the notation $a \leftrightsquigarrow b$ to mean that a and b share an edge in the strong top-graph. Consider a strongly top-connected domain $\mathbb{D}$ and three alternatives a, b, and c such that $a \leftrightsquigarrow b$ and $b \leftrightsquigarrow c$. Consider the preferences $\bar{R}^a \equiv ab\cdots$ and $\bar{R}^b \equiv ba\cdots$ such that they agree on the ranking of the alternatives other than a and b. Let $\bar{R}^c$ be arbitrary. Then, we have $U(c, \overline{R}^a) = U(c, \overline{R}^b)$, and thereby $U(c, \overline{R}^a) \cap U(a, \overline{R}^c) \subseteq U(c, \overline{R}^a) = U(c, \overline{R}^b) \subseteq U(a, \overline{R}^b) \cup U(c, \overline{R}^b)$, implying the richness property.

REMARK 2.2. A domain is locally connected if for every two preferences in that domain, there is a sequence of preferences starting from the former and ending at the latter such that every two consecutive preferences in that sequence differ by the ordering of exactly two (consecutively ranked) alternatives. It is left to the reader to verify that every locally connected domain satisfies the richness property.

A domain is said to satisfy Condition 1 if it satisfies top-connectedness, pervasiveness, and richness. Condition 1 is a mild technical condition. Most domains of practical importance, such as the unrestricted domain, single-peaked domains, single-crossing domains, single-dipped domains, circular domains (Sato (2010)), etc., satisfy this.

2.3 Properties of choice functions

In this section, we present some properties of choice functions. Let φ be a choice function from $\mathbb{D}^N$ to A. The choice function φ is *unanimous* if, whenever all the agents agree on their preferences, the best alternative of that common preference is chosen. More formally, φ is unanimous if for all profiles p such that $p(i) = R$ for all $i \in N$ and some $R \in \mathbb{D}$, the outcome $\varphi(p)$ is the best alternative of R.[3] The choice function φ is *Pareto optimal* if its outcome is not Pareto dominated at any profile, that is, for all profiles p, there is no alternative $a \in A$ such that $p(i) \equiv \cdots a \cdots \varphi(p) \cdots$ for all agents i. The choice function φ is called *dictatorial* with *dictator* i if for all profiles p, $\varphi(p)$ is the best alternative of agent i. The choice function φ is called *anonymous* if it is symmetric in its arguments. The choice function φ is *strategy-proof* if no agent can manipulate it, that is, can bend its outcome in his/her favor by misreporting his/her sincere preference. More formally, φ is strategy-proof if for all agents i and all profiles p and q with $p(j) = p(j)$ for all agents $j \in N \setminus \{i\}$, we have either $\varphi(p) = \varphi(q)$ or $p(i) \equiv \cdots \varphi(p) \cdots \varphi(q) \cdots$. The choice function φ is *intermediate strategy-proof* if for all $S \subseteq N$, all $R \in \mathbb{D}$, and all profiles p and q such that $p = (R^S, p|_{N\setminus S})$ and $p|_{N\setminus S} = q|_{N\setminus S}$, we have either $\varphi(p) = \varphi(q)$ or $R \equiv \cdots \varphi(p) \cdots \varphi(q) \cdots$. In words, intermediate strategy-proofness ensures that any group of agents (coalition) who have the same preference at a profile, cannot manipulate the outcome by misreporting in any arbitrary manner. The choice function φ is *Maskin monotone* (Maskin (1979)) if $\varphi(p) = \varphi(q)$ for all profiles p and q such that $U(\varphi(p), q(i)) \subseteq U(\varphi(p), p(i))$ for all agents i.

REMARK 2.3. In Peters et al. (1991), it is shown that strategy-proofness and intermediate strategy-proofness are equivalent on any domain. Because of this equivalence, we refer to the latter by the former. Furthermore, it is well-known that strategy-proofness implies (Maskin) monotonicity. We use these facts in our proofs (without any further reference).

[3]Here, unanimity is slightly weaker than the usual notion of the same defined as follows: $\varphi(p) = a$ whenever $p(i) \in D^a$ for all agents i. It can be seen that under strategy-proofness (to be defined later), these two formulations are equivalent.

3 Top-connected possibility domains

We say that a domain $\mathbb{D}$ is a *possibility domain* if there exist non-dictatorial, unanimous, and strategy-proof choice functions φ from $\mathbb{D}^N$ to A. In this section, we show that a domain satisfying Condition 1 is a possibility domain if and only if it contains an inseparable top-pair.

The following example establishes the if part of the above-mentioned theorem. More formally, it shows that if a domain contains an inseparable top-pair, then there is a non-dictatorial, unanimous, and strategy-proof choice function on it.

Example 3.1. Consider a domain $\mathbb{D}$. Suppose that for two alternatives a and b, the pair ab is an inseparable top-pair in $\tau(\mathbb{D})$. Define the choice function φ^{ab} for an arbitrary profile p in $\mathbb{D}^N$ as follows

$$\begin{aligned} \varphi^{ab}(p) &= c \text{ if } p(1) \equiv c \cdots \text{ and } c \neq a \\ &= a \text{ if } p(1) \equiv a \cdots \text{ and } ab \in p(2) \\ &= b \text{ if } p(1) \equiv a \cdots \text{ and } ba \in p(2). \end{aligned}$$

Clearly, φ^{ab} is non-dictatorial and unanimous. It is even Pareto optimal. In the following, we argue that it is strategy-proof. Agents $i \geq 3$ cannot manipulate as they have no influence on the outcome at any profile. Agent 2 can determine the outcome only at the profiles where a is the best alternative of agent 1's preference. At any such profile, the choice function selects agent 2's most preferred alternative from the set $\{a, b\}$. Therefore, 2 cannot manipulate the choice function. Now, consider agent 1. Note that the choice function chooses his/her best alternative except from the situations when his/her best alternative is a and agent 2 prefers b to a, in which case it choses b. Further note that by the construction of φ^{ab}, its outcome always lies in $\tau(\mathbb{D})$. Since ab is an inseparable top-pair in $\mathbb{D}$, whenever a is the best alternative in agent 1's preference, b is the second-best among the alternatives in $\tau(\mathbb{D})$. So, the only way agent 1 can manipulate in such a situation is by making a the outcome. However, that is not possible by the construction of φ^{ab} since agent 2 prefers a to b. So, agent 1 also cannot manipulate.

The following theorem shows that existence of an inseparable top-pair is also necessary for a domain satisfying Condition 1 to be a possibility domain.

Theorem 1. *Let a domain $\mathbb{D}$ satisfy Condition 1. Then there exist non-dictatorial, unanimous, and strategy-proof choice functions on $\mathbb{D}$ if and only if it has an inseparable top-pair in $\tau(\mathbb{D})$.*

The proof of this theorem is relegated to Appendix A.

In what follows, we provide an example to show that the richness condition is necessary for Theorem 1.

Example 3.2 (*Choosing corner points of a square under separable preferences*)**.** Let a, b, c, and d be the corners of a square in $\mathbb{R}^2$ with the coordinates $(-1,1)$, $(1,1)$, $(1,-1)$, and $(-1,-1)$, respectively.[4] Consider the domain $\mathbb{D} = \{abdc, bacd, bcad, cbda, cdba, dcab, dacb, adbc\}$. Note that $\mathbb{D}$ is top-connected having no inseparable top-pair but it does not satisfy the richness property of Condition 1. One can construct a unanimous and strategy-proof choice function on $\mathbb{D}$ by composing two (independent) choice functions one for each dimension. In other words, one can use a unanimous and strategy-proof choice function to select between top and bottom, and another unanimous and strategy-proof choice function (independently) to select between left and right, and can decide the final outcome by combining these two outcomes. For instance, if top is selected by the first choice function and right is selected by the second, then the final outcome is (top-left), which means $(-1,1)$. Note that for odd number of agents, such choice functions can be made Pareto optimal (by using a majority rule for both the dimensions). Therefore, unanimity cannot be replaced by Pareto optimality in Proposition 3.1 of Aswal et al. (2003) (or in Lemma 5 in Appendix A).

Now, we proceed to present our next result. It is important to note that although the choice function presented in Example 3.1 is non-dictatorial, it is far from being anonymous. In fact, for this choice function, agent 1 decides the outcome for all but one type of profiles. In view of this, in the following theorem we provide a necessary condition for a domain to admit anonymous, unanimous, and strategy-proof choice functions. In Corollary 2, we present a collection of domains satisfying Condition 1 for which this necessary condition is also sufficient.

Theorem 2. *Let a domain $\mathbb{D}$ satisfy Condition 1. Suppose $\mathbb{D}$ has a top-circuit. Then, there does not exist an anonymous, unanimous, and strategy-proof choice function on $\mathbb{D}$.*

The proof of this theorem is relegated to Appendix A.

4 Applications

In this section, we present a few applications of Theorems 1 and 2.

4.1 Single-peaked Euclidean domains

Let $\mathcal{E}$ be a finite dimensional Euclidean space with Euclidean norm $||\ldots||$. Suppose that the set of alternatives A is a finite subset of $\mathcal{E}$. If $\mathcal{E}$ is two-dimensional (i.e., a Euclidean plane), the alternatives can be interpreted as potential geographical locations for developing a public facility like a hospital, town

[4]The domain $\mathbb{D}$ can be obtained as follows: Consider Euclidean preferences with a bliss point, say x, such that preference decreases with Euclidean distance from x. Consider a domain that consists of all Euclidean preferences for which the bliss point is not on the perpendicular bisector of any side (in other words, not on any of the axes X or Y). Thus, all the preferences in the domain are strict. Then, this domain will contain the preferences $abdc$, $bacd$, $bcad$, $cbda$, $cdba$, $dcab$, $dacb$, and $adbc$.

hall, school, sports facility, etc. For arbitrary dimensions, the alternatives can be thought of as budgetary proposals, where each dimension represents a public issue like education, defense, health care, etc.

In such collective decision problems, Euclidean distance gives an estimate of the proximity of the alternatives, and hence can be used to describe the structure of the preferences. Such preferences are called *Euclidean orders*. For such a preference, there is a *bliss* point, say u, such that an alternative a in A is weakly preferred to another alternative b in A if $||u - a|| \leq ||u - b||$, that is, if a is at least as close to u as b is. We denote such a weak order[5] by E_u. For such an order, alternatives that are at an equal distance from u are in the same indifference class. We denote these indifference classes by C_1, C_2, etc. We write $E_u = C_1 C_2 \cdots C_k$ to denote the preference having indifference classes as $C_1, \cdots, C_k$ such that each alternative in C_s is strictly preferred to each alternative in C_t for all $1 \leq s < t \leq k$.

Consider the situation where the set of all possible bliss points is the convex hull of A, denoted by $convexhull(A)$. Note that a bliss point need not be a possible location and an alternative can be seen as a compromised location of some bliss points. Consider the domain $\mathbb{D}$ consisting of the set of all Euclidean linear orders on A, that is, $\mathbb{D} = \{R \in \mathbb{L} : R \subseteq E_u \text{ for some } u \in convexhull(A)\}$. In other words, $\mathbb{D}$ contains all linear extensions (that is, where indifferences are broken arbitrarily) of every Euclidean order E_u with bliss point u in the convex hull of A. Note that since any point in A can be a bliss point, we have $\tau(\mathbb{D}) = A$. Throughout this section, we call such a domain a single-peaked Euclidean domain.

The following claim shows that every single-peaked Euclidean domain satisfies Condition 1.

Claim 1. *Every single-peaked Euclidean domain $\mathbb{D}$ satisfies Condition 1.*

The proof of this claim is relegated to Appendix B.

Let $\mathbb{D}$ be a single-peaked Euclidean domain. Consider the top-graph of $\mathbb{D}$, say $\mathcal{G}$. Recall that by the definition of a top-graph, there is an edge between a and b in $\mathcal{G}$ if and only a is top-connected to b. We say that a pair of vertices ab form a *loose end* in $\mathcal{G}$ if the edge between a and b is the *only* edge from a in $\mathcal{G}$. Note that ab forms a loose end in $\mathcal{G}$ if and only if ab is an inseparable top-pair in $\mathbb{D}$. Therefore, we obtain the following corollary from Theorem 1.

Corollary 1. *A single-peaked Euclidean domain is a possibility domain if and only if its top-graph has a loose end.*

Our next corollary presents a characterization of all single-peaked Euclidean domains that admit choice functions that are anonymous as well as unanimous and strategy-proof.

Corollary 2. *There exist anonymous, unanimous, and strategy-proof choice functions on a single-peaked Euclidean domain if and only if it has no top-circuits.*

[5] A weak order is a reflexive, complete, and transitive binary relation on A.

The proof of this corollary is relegated to Appendix B.

In what follows, we provide two examples where we apply Corollaries 1 and 2 to deduce the anonymous, unanimous, and strategy-proof choice functions on some single-peaked Euclidean domains. Our first example deals with three alternatives, whereas the second one deals with four.

Example 4.1 (Three locations in $\mathcal{E}$)**.** Suppose that $A = \{a, b, c\}$ is a set of three points in the Euclidean plane and $\mathbb{D}$ is the single-peaked Euclidean domain over A. Let us denote by $triangle(a, b, c)$ the triangle with corners a, b, and c. By using basic geometry, it follows that if the circumcenter of $triangle(a, b, c)$ (i.e., the center of the circumscribed circle of $triangle(a, b, c)$) lies within $triangle(a, b, c)$, then all six (strict) preferences on A are in $\mathbb{D}$. Since this happens when $triangle(a, b, c)$ is a right-angled triangle or an acute-angled triangle, it follows that $\mathbb{D}$ is an impossibility domain in such cases. Suppose that $triangle(a, b, c)$ is an obtuse-angled triangle with the angle $\angle abc$ being obtuse. Then, $\mathbb{D} = \{abc, bac, bca, cba\}$, and the top-graph $\mathcal{G}$ of $\mathbb{D}$ is a line which can be pictured as follows

$$a \leftrightsquigarrow b \leftrightsquigarrow c.$$

By Corollary 2, this means $\mathbb{D}$ admits anonymous, unanimous, and strategy-proof choice functions.

Combining all these observations, it follows that $\mathbb{D}$ is a possibility domain if and only if $triangle(a, b, c)$ is an obtuse triangle.

Next, we analyze the case of four or more locations in the Euclidean plane. A detailed analysis as in Example 4.1 is cumbersome for such cases. Therefore, we only provide a sketch of how such cases can be treated.

Example 4.2 (Four locations in $\mathcal{E}$)**.** Suppose that $A = \{a, b, c, d\}$ is a set of four points in the Euclidean plane and $\mathbb{D}$ the single-peaked Euclidean domain over A. Let $\mathcal{G}$ be the top-graph of $\mathbb{D}$.

Note that if $\mathcal{G}$ contains a top-circuit, say $a \leftrightsquigarrow b \leftrightsquigarrow c \leftrightsquigarrow d \leftrightsquigarrow a$, which will happen, for instance, if the points $\{a, b, c, d\}$ are corners of a squire, then by Corollary 1, $\mathbb{D}$ is an impossibility domain. On the other hand, if the quadrangle with corners a, b, c, and d is such that the circumcenters of $triangle(a, b, c)$, $triangle(b, c, d)$, $triangle(c, d, a)$, and $triangle(d, a, b)$ are outside the convex hull of A, then $\mathcal{G}$ is a line graph which can be pictured as

$$a \leftrightsquigarrow b \leftrightsquigarrow c \leftrightsquigarrow d.$$

By Corollary 2, it follows that $\mathbb{D}$ admits anonymous, unanimous, and strategy-proof choice functions.

The above two cases are two extreme cases. Now, consider the case where a, b, and c are the corners of an equilateral triangle and d is located outside this triangle such that d is top-connected to only c. So, the circumcenters of $triangle(d, a, b)$ and $triangle(b, c, d)$ are not in the convex hull of A. This means dc is a loose end in $\mathcal{G}$ and by Corollary 1, it follows that $\mathbb{D}$ is a possibility domain. We conjecture that in this case the choice function defined in Example 3.1 is the only non-dictatorial, unanimous, and strategy-proof choice function on this domain.

Finally, consider the case where a, b, and c are the corners of an equilateral triangle and d is located on the circumcenter of this triangle. Then, the edge set of $\mathcal{G}$ consists of the edges $a \leftrightsquigarrow d$, $b \leftrightsquigarrow d$, and $c \leftrightsquigarrow d$. By Corollary 2, it follows that $\mathbb{D}$ admits anonymous, unanimous, and strategy-proof choice functions.

It is worth noting that the four types of graphs discussed in Example 4.2 are the *only* four types that can occur with four locations in the Euclidean plane. It is tedious to describe what type of conditions on locations will lead to which graph. This is a problem of geometry, and therefore is omitted here.

4.2 Single-dipped Euclidean domains on convex polytopes

In this subsection, we consider single-dipped preferences on convex polytopes. A convex polytope is the convex hull of a finite number of points in a finite dimensional Euclidean space $\mathcal{E}$. A single-dipped preference is one for which preference increases as one goes far away from the worst alternative of the preference. Such preferences arise in situations where a public bad such as a nuclear plant, windmill, garbage dump, etc., has to be located or in situations where pure commodities are preferred to the mixtures of commodities.

Let the set of alternatives A be a convex polytope. A *single-dipped Euclidean order* on A with dip at u in A, denoted by $-E_u$, is defined as follows: $-E_u = \{ab : ||a-u|| \geq ||b-u||\}$. A *single-dipped Euclidean domain on* A is defined as $\mathbb{D} = \{R \in \mathbb{L} : R \subseteq -E_u \text{ for some } u \in A\}$. Note that these preferences are strict by definition, which in particular means that the relative ordering of the alternatives that are at an equal distance from the dip is unrestricted. Clearly, for such a domain $\mathbb{D}$, only some extreme points of A constitute the set $\tau(\mathbb{D})$: the set of alternatives that are the best at some preference in $\mathbb{D}$. It is worth noting that all extreme points of A need not be in $\tau(\mathbb{D})$. For instance, if A is the convex hull of a triangle, say *triangle*(a,b,c), such that $\angle abc$ is obtuse, then $\tau(\mathbb{D}) = \{a,c\}$. By definition, $\tau(\mathbb{D})$ is finite. Further note that although A has finitely many extreme points, it is not finite.

The following claim shows that a single-dipped Euclidean domain on a convex polytope satisfies Condition 1.

Claim 2. *Every single-dipped Euclidean domain on a convex polytope satisfies Condition 1.*

The proof of this claim is relegated to Appendix B.

Next, we characterize all single-dipped Euclidean domains on convex polytopes that have an inseparable top-pair.

Claim 3. *Let $\mathbb{D}$ be a single-dipped Euclidean domain. Suppose that a and b are two different alternatives in $\tau(\mathbb{D})$. Then ab is an inseparable top-pair in $\mathbb{D}$ if and only if $\tau(\mathbb{D}) = \{a, b\}$.*

The proof of this claim is relegated to Appendix B.

Note that if a domain $\mathbb{D}$ has no top-circuit, then it must have an inseparable top-pair. Therefore, Theorems 1 and 2 imply the following.

Corollary 3. *Let A be a convex polytope in $\mathcal{E}$ and let $\mathbb{D}$ be a single-dipped Euclidean domain on A. Then the following three statements are equivalent:*

(i) There exist non-dictatorial, unanimous, and strategy-proof choice functions from $\mathbb{D}^N$ to A.

(ii) $\tau(\mathbb{D})$ consists of exactly two alternatives.

(iii) There exist anonymous, unanimous, and strategy-proof choice functions from $\mathbb{D}^N$ to A.

The above results clarify that non-dictatorship (under unanimity and strategy-proofness) can be achieved for very specific cases. A similar result for a smaller set of preferences can also be found in Öztürk et al. (2014), however, in that paper, the analysis is restricted to the Euclidean plane only.

4.3 Strategic preference aggregation problem

In this section, we consider the standard preference aggregation problem where the preferences of the the individuals in a society are to be aggregated to a collective preference. Such situations occur when a committee has to prepare a ranking/list of the applicants for a vacancy or of the contestants in a competition, etc., based on the preferences (or judgments) of the members of the committee.

A list is a linear order on the set of alternatives A. A preference function ψ (also known as welfare function) on a domain $\mathbb{D}$ assigns a linear order in $\mathbb{L}$ at every profile of linear orders p in $\mathbb{D}^N$. The Kemeny distance between two linear orders R^1 and R^2 is defined as $\delta(R^1, R^2) = \frac{1}{2}\#(R^1 \triangle R^2)$, where $R^1 \triangle R^2$ denotes the symmetric difference between R^1 and R^2.[6] The notion of *strategy-proofness* can be defined for preference rules by means of Kemeny distance δ in the following way: A preference rule ψ is strategy-proof if for all agents i and all profiles p and q with $p(j) = q(j)$ for all $j \in N \setminus \{i\}$, we have

$$\text{either } \delta(p(i), \psi(p)) < \delta(p(i), \psi(q)) \text{ or } \psi(p) = \psi(q).$$

Strategy-proofness implies that if an agent misreports his/her preference, then either the outcome does not change or it goes farther away (with respect to the Kemeny distance) from his/her sincere preference.

[6]The symmetric difference between R^1 and R^2 is the set of pairs ab such that either $ab \in R^1$ and $ab \notin R^2$, or $ab \in R^2$ and $ab \notin R^1$.

Bossert and Sprumont (2014) consider the same problem, however their notion of strategy-proofness is slightly different from that of ours. In their paper, an agent manipulates if the outcome at the misreported profile lies "strictly between" his/her sincere preference and the original outcome. Note that in such cases, the Kemeny distance between his/her sincere preference and the manipulated outcome will be lesser than that between his/her sincere preference and the original outcome. However, having a lesser Kemeny distance does not necessarily mean that the manipulated outcome will be strictly between his/her sincere preference and the original outcome. Thus, if a rule is manipulable according to the notion in Bossert and Sprumont (2014), then it will also be manipulable according to our notion. However, the converse does not hold.

Preference rule ψ is called *unanimous* if $\psi(p) = R$ at any unanimous profile p such that $p(i) = R$ for all agents i in N.

Consider a preference $p(i)$ on A. An extension of $p(i)$ to a preference $\widetilde{p}(i)$ on the linear orders over $\mathbb{L}$ is defined as follows: for all R^1 and R^2 in $\mathbb{L}$

$$R^1 \widetilde{p}(i) R^2 \text{ if } \delta(p(i), R^1) < \delta(p(i), R^2).$$

In other words, $\widetilde{p}(i)$ is a linear order on $\mathbb{L}$ such that $p(i)$ is the most preferred list at $\widetilde{p}(i)$, and preference decreases as the Kemeny distance from $p(i)$ increases. For a domain $\mathbb{D}$, define $\widetilde{\mathbb{D}}$ as the set of all possible extensions $\widetilde{p}(i)$ of the linear orders $p(i)$ in $\mathbb{D}$.

The problem of studying the unanimous and strategy-proof preference function ψ can be translated to that of studying a particular type of rules $\widetilde{\varphi}$ from $\widetilde{\mathbb{D}}^N$ to $\mathbb{L}$ in the following way. For every preference function ψ, define the rule $\widetilde{\varphi}^{\psi}$ from $\widetilde{\mathbb{D}}^N$ to $\mathbb{L}$ as follows: for all profiles $\widetilde{p}$ in $\widetilde{\mathbb{D}}$,

$$\widetilde{\varphi}^{\psi}(\widetilde{p}) = \psi(p),$$

where for all agents i in N, $p(i)$ is the most preferred list of $\widetilde{p}(i)$.

It is straightforward to prove that if ψ is unanimous and strategy-proof, then so is $\widetilde{\varphi}^{\psi}$. Additionally, if ψ is anonymous, then $\widetilde{\varphi}^{\psi}$ is also anonymous.

It is worth noting that even if $\mathbb{D}$ is unrestricted, that is, $\mathbb{D} = \mathbb{L}$, the domain $\widetilde{\mathbb{D}}$ is restricted. For instance, if A contains 3 alternatives, then each of the six possible preferences $p(i)$ in $\mathbb{L}$ can be extended to 4 different preferences in $\widetilde{\mathbb{D}}$. So, in total $\widetilde{\mathbb{D}}$ will consist of 24 orders on lists. However, as there are six lists in $\mathbb{D}$, the unrestricted domain on $\mathbb{D}$ will have $6! = 720$ linear orders. In case of four applicants, these numbers are 40608 and 24!, respectively.

Suppose $\mathbb{D} = \mathbb{L}$. It follows straightforwardly that $\widetilde{\mathbb{D}}$ satisfies Condition 1 and that it has no inseparable top-pair. Consider a unanimous and strategy-proof preference function ψ. Then, $\widetilde{\varphi}^{\psi}$ is also unanimous and strategy-proof, and hence by Theorem 1, $\widetilde{\varphi}^{\psi}$ is dictatorial, i.e., there is a dictator, say j, in N such that for all profiles $\widetilde{p}$ in $\widetilde{\mathbb{D}}^N$

$$\widetilde{\varphi}^{\psi}(\widetilde{p}) = p(j).$$

By construction, this implies that ψ is also dictatorial with dictator j.

A similar result can be found in Bossert and Storcken (1992). However, their model and assumptions are slightly different from the one presented here.

5 Conclusion

In this paper, we have imposed some additional structure on domains and have investigated when such domains are possibility. In Theorem 1, we present a necessary and sufficient condition for such domains to be possibility domains. We demonstrate by means of examples that this theorem applies to a wide range of domains. We have demonstrated by means of an example that a possibility domain need not guarantee a choice function that distributes the decisive power more or less evenly amongst the agents. Consequently, we have provided a necessary condition for a domain to admit anonymous, unanimous, and strategy-proof choice functions in Theorem 2.

To the best of our knowledge, the results that we have obtained on choosing from a finite number of locations in a Euclidean space are new to the literature. In a related paper, Zhou (1991) considers the problem where the set of alternatives is a convex subset (and hence infinite) of the Euclidean space. He shows that there are no non-dictatorial, unanimous, and strategy-proof choice functions on such domains. Demange (1982) considers the problem of single-peaked domains on graphs. He provides a sufficient condition for a domain to admit choice functions based on (pairwise) majority comparison. On the other hand, we have shown in Theorem 2 that for any domain satisfying Condition 1, tree structure in the top-graph is necessary for admitting anonymous, unanimous, and strategy-proof choice functions.

Theorem 1 is closely related to the literature on linked domains (Aswal et al. (2003)), β and γ domains (Pramanik (2015)), and the circular domains (Sato (2010)). However, for all these results, the domains are assumed to be minimally rich (or regular), which, according to our notation, means $A = \tau(\mathbb{D})$. On the other hand, we impose certain conditions (formulated as Condition 1) on the domains, which they do not impose. To show the independence of Theorem 1 from these results, let us consider a domain $\mathbb{D}$ such that $A = \tau(\mathbb{D}) = \{x_1, \ldots, x_m\}$, x_t is only top-connected to both x_{t+1} and x_{t-1} for all $1 < t < m$, and x_m is top-connected to x_1. So, the top-graph of this domain contains a cycle (containing all the alternatives). Suppose that the domain is strongly top-connected and $\mathbb{D}$ is a minimal domain satisfying these conditions. It can be verified that for $m \geq 4$, this is not a linked domain as in Aswal et al. (2003), or a β or γ domain as in Pramanik (2015), or a circular domain as in Sato (2010). So, these results do not apply to the domain $\mathbb{D}$. However, Theorem 1 says that it is an impossibility domain. Conversely, one can find domains that do not satisfy Condition 1 but on which these existing results apply. In is worth mentioning that the objective of these papers is to find sufficient conditions for a domain to be impossibility, whereas we strive for necessary and sufficient conditions for the same.

Appendix A

Let V be a set of S-unanimous profiles in $\mathbb{D}^N$. Given a choice function φ, we say that a coalition (a subset of N) S is *decisive on* V if $\varphi(R^S, p|_{N \setminus S}) = a$ for all $(R^S, p|_{N \setminus S}) \in V$ with $R \in \mathbb{D}^a$. A coalition S is said to be *decisive* if it is decisive on the set of all S-unanimous profiles in $\mathbb{D}^N$. We say that a choice function φ is *alternative decisive* if for all coalitions S, either S is decisive or $N \setminus S$ is decisive.

A major step in proving that some domain is an impossibility is to show that the decisiveness of coalitions at some profiles spreads "epidemically" to all possible profiles. If the domain is unrestricted, then such epidemic spread can be established without much effort. However, for the restricted domains that we consider in this paper, subtle techniques are needed to show this step.

We show that the decisiveness of a coalition disseminates along a path of top-connected alternatives. To show this, first we show that if a coalition is decisive at profiles where the other agents are unanimous, then it is decisive at all profiles. For unrestricted domains, this follows directly from Maskin monotonicity (Maskin (1979)). Here, some extra work is needed.

Lemma 1. *Let φ be a unanimous and strategy-proof choice function from $\mathbb{D}^N$ to A. Suppose that a coalition S is decisive on all $(S, N \setminus S)$-unanimous profiles. Then S is decisive.*[7]

Proof. Let p be an S-unanimous profile such that $p(i) = R$ for all i in S, where $R \in \mathbb{D}^a$ for some $a \in A$. It is sufficient to show that $\varphi(p) = a$. We prove this by using induction on the number of different preferences at the profile p. Let $k = \#\{p(j) : j \in N\}$ be the number of different preferences at p. The case where $k = 1$ follows from unanimity. Moreover, the base case $k = 2$ follows from the fact that S is decisive on all $(S, N \setminus S)$-unanimous profiles. We proceed to prove the induction step. Suppose that the lemma holds for all profiles at which the number of different preferences is k. Assume that the number of different preferences at p is $k+1$. If $\varphi(p) = a$, then we are done. So, suppose $\varphi(p) \neq a$. Let $T_1, T_2, \ldots, T_{k+1}$ be the partition of N such that $S \subseteq T_1$, and for all $l \in \{1, 2, \ldots, k\}$, all the agents in T_l have the same preference, say R^l. As $k \geq 2$, we have $k + 1 \geq 3$. Let q be the profile such that $q(i) = p(i)$ if $i \notin T_2$ and $q(i) = R^3$ if

[7] This lemma holds for any arbitrary domain $\mathbb{D}$.

$i \in T_2$. By the induction hypothesis, $\varphi(q) = a$. Strategy-proofness implies that $\varphi(p)a \in R^2$. Now, consider the profile r such that $r(i) = p(i)$ if $i \notin T_3$ and $r(i) = R^2$ if $i \in T_3$. By the induction hypothesis, $\varphi(r) = a$. Strategy-proofness now implies that $a\varphi(p) \in R^2$. However, the facts that $\varphi(p) \neq a$, $\varphi(p)a \in R^2$, and $a\varphi(p) \in R^2$ contradict that R^2 is a strict order. Hence, $\varphi(p) = a$. ■

The following three lemmas prove that decisiveness spreads along a path of top-connected alternatives. The proof is based on induction. Lemmas 2 and 3 consider the base case and Lemma 4 establishes the inductions step.

Lemma 2. *Let a and b be directly top-connected alternatives and let S be a coalition. Suppose that φ is a unanimous and strategy-proof choice function from $\mathbb{D}^N$ to A. Then either S is decisive on all ab-$(S, N \setminus S)$-unanimous profiles or $N \setminus S$ is decisive on all ab-$(S, N \setminus S)$ -unanimous profiles.*

Proof. As a and b are directly top-connected, there are preferences $R^{ab} \equiv ab \cdots$ in $\mathbb{D}^a$ and $R^{ba} \equiv ba \cdots$ in $\mathbb{D}^b$. Consider the profile $((R^{ab})^S, (R^{ba})^{N \setminus S})$ in $(\mathbb{D}^a)^S \times (\mathbb{D}^b)^{N \setminus S}$. Because of unanimity, $\varphi((R^{ab})^N) = a$. Therefore, strategy-proofness implies $\varphi((R^{ab})^S, (R^{ba})^{N \setminus S}) \in \{a, b\}$. Without loss of generality, assume $\varphi((R^{ab})^S, (R^{ba})^{N \setminus S}) = a$. It is sufficient to prove that S is decisive on all ab-$(S, N \setminus S)$-unanimous profiles. Let $R^1 \in \mathbb{D}^a$ and $R^2 \in \mathbb{D}^b$. It is sufficient to prove that $\varphi((R^1)^S, (R^2)^{N \setminus S}) = a$. First we prove $\varphi((R^{ab})^S, (R^2)^{N \setminus S}) =$ a. Because of unanimity, $\varphi((R^2)^N) = b$. This, together with strategy-proofness, implies $\varphi((R^{ab})^S, (R^2)^{N \setminus S}) \in$ $\{a, b\}$. Because $\varphi((R^{ab})^S, (R^{ba})^{N \setminus S}) = a$, strategy-proofness implies $\varphi((R^{ab})^S, (R^2)^{N \setminus S}) \neq b$. So, $\varphi((R^{ab})^S, (R^2)^{N \setminus S})$ a. Now, by means of monotonicity, we have $\varphi((R^1)^S, (R^2)^{N \setminus S}) = a$. ■

Lemma 3. *Let $a \leftrightsquigarrow b$ and $b \leftrightsquigarrow c$, where a, b and c are three different alternatives and let S be a coalition. Suppose that φ is a unanimous and strategy-proof choice function from $\mathbb{D}^N$ to A. Suppose further that S is decisive on all ab-$(S, N \setminus S)$-unanimous profiles. Then, S is decisive on all bc-$(S, N \setminus S)$-unanimous profiles.*

Proof. Assume for contradiction that S is not decisive on all bc-$(S, N \setminus S)$-unanimous profiles. Lemma 2 implies that $N \setminus S$ is decisive on all bc-$(S, N \setminus S)$-unanimous profiles. Consider the profile $((R^a)^S, (R^c)^{N \setminus S})$. Suppose $\varphi((R^a)^S, (R^c)^{N \setminus S}) = d$.

First, we show that d in $U(c, R^a) \cap U(a, R^c)$. By unanimity, $\varphi((R^c)^N) = c$. Therefore, by strategy-proofness, d is in $U(c, R^a)$. By a similar logic, it can be shown that d is in $U(a, R^c)$. Hence, d is in $U(c, R^a) \cap U(a, R^c)$.

Next, we show $d \neq c$ and $d \neq a$. Suppose $d = c$. Because $b \leftrightsquigarrow c$, there is a preference $R^{bc} \equiv bc \cdots$ in $\mathbb{D}^b$. As S is decisive on all ab-$(S, N \setminus S)$-unanimous profiles, $\varphi((R^a)^S, (R^{bc})^{N \setminus S}) = a$. However, as $\varphi((R^a)^S, (R^c)^{N \setminus S}) = c$ and c is strictly preferred to a at R^{bc}, this contradicts strategy-proofness at $((R^a)^S, (R^{bc})^{N \setminus S})$. So, $d \neq c$. Similarly, one can show $d \neq a$.

By the richness assumption of Condition 1, there are R^a in $\mathbb{D}^a$ and R^c in $\mathbb{D}^c$ such that for all x in $U(c, R^a) \cap U(a, R^c)$, there are R^b in $\mathbb{D}^b$ with $x \in U(c, R^b) \cup U(a, R^b)$. So, for some R^b in $\mathbb{D}^b$, we have $d \in U(c, R^b) \cup U(a, R^b)$. Without loss of generality, let $d \in U(c, R^b)$. As $N \setminus S$ is decisive on all bc-$(S, N \setminus S)$-unanimous profiles, we have $\varphi((R^b)^S, (R^c)^{N \setminus S}) = c$. However, since $\varphi((R^a)^S, (R^c)^{N \setminus S}) = d$ and d is strictly preferred to c at R^b, this means strategy-proofness is violated at $((R^b)^S, (R^c)^{N \setminus S})$. ■

Lemma 4. *Let φ be a unanimous and strategy-proof choice function from $\mathbb{D}^N$ to A and let S be a coalition. Suppose that the alternatives $x_1, x_2, \ldots, x_k$, where $k \geq 3$, are such that*

(i) $\#\{x_{t-1}, x_t, x_{t+1}\} = 3$ *for all* $t \in \{2, 3, \ldots, k-1\}$,

(ii) $x_t \leftrightsquigarrow x_{t+1}$ *for all* $t \in \{1, 2, \ldots, k-1\}$, *and*

(iii) *S is decisive on all $x_t x_{t+1}$-$(S, N \setminus S)$-unanimous profiles for all $t \in \{1, 2, \ldots, k-1\}$.*

Then, S is decisive on $x_t x_s$-$(S, N \setminus S)$-unanimous profiles for all t, s such that $1 \leq t < s \leq k$.

Proof. For ease of presentation, let us denote an arbitrary subsequence of m, where $2 \leq m \leq k$, consecutive elements of $x_1, x_2, \ldots, x_k$ as $y_1, y_2, \ldots, y_m$. We prove the lemma by using induction on the length m of such subsequences.

Base case: For subsequences of length 2, it follows from the assumption (iii) of the lemma that S is decisive on $y_1 y_2$-$(S, N \setminus S)$-unanimous profiles.

Induction step: Suppose that for any subsequence $y_1, y_2, \cdots, y_m$ of length $m; m < k$, S is decisive on all $y_t y_s - (S, N \setminus S)$-unanimous profiles for all $t, s \in \{1, \ldots, m\}$ with $t < s$. We show that the same holds for any subsequence of length $m+1$, that is, for any subsequence $y_1, y_2, \cdots, y_{m+1}$, S is decisive on $y_t y_s$-$(S, N \setminus S)$-unanimous profiles for all $t, s \in \{1, \ldots, m+1\}$ with $t < s$. To prove this induction step, it suffices to show that S is decisive on $y_1 y_{m+1}$-$(S, N \setminus S)$-unanimous profiles. We distinguish the following two cases.

Case 1. Suppose $y_1 = y_{m+1}$. However, then a $y_1 y_{m+1}$-$(S, N \setminus S)$-unanimous profile becomes a $y_1 y_1$-$(S, N \setminus S)$-unanimous profile, and hence S is decisive in a trivial manner.

Case 2. Suppose $y_1 \neq y_{m+1}$. Let $(R^S, \overline{R}^{N \setminus S})$ be the $y_1 y_{m+1}$-$(S, N \setminus S)$-unanimous profile such that $R \equiv y_1 y_2 \cdots$. In view of monotonicity, it is sufficient to prove that $\varphi(R^S, \overline{R}^{N \setminus S}) = y_1$. Consider $\widetilde{R} \equiv y_2 y_1 \cdots$ in $\mathbb{D}^{y_2}$. By applying the induction hypothesis to the subsequence $y_2, y_3, \cdots, y_{m+1}$, we have $\varphi(\widetilde{R}^S, \overline{R}^{N \setminus S}) = y_2$. As $R \equiv y_1 y_2 \cdots$, strategy-proofness implies $\varphi(R^S, \overline{R}^{N \setminus S}) \in \{y_1, y_2\}$. Again, by the induction hypothesis, $\varphi(R^S, (\widetilde{R})^{N \setminus S}) = y_1$. This, together with monotonicity, implies $\varphi(R^S, \overline{R}^{N \setminus S}) \neq y_2$. So, $\varphi(R^S, \overline{R}^{N \setminus S}) = y_1$. ■

In the following lemma, we establish a crucial step to deduce the impossibility theorems. In this step, for arbitrary non-empty subset N' of N, we define choice functions φ' from $\mathbb{D}^{N'}$ to A. It is straightforward to define unanimity, strategy-proofness, and alternative decisiveness for such choice functions, and therefore these details are omitted here. The following lemma establishes that it is sufficient to show that every unanimous and strategy-proof choice function φ' from $\mathbb{D}^{N'}$ to A is alternative decisive to prove that the domain $\mathbb{D}$ is impossibility.

Lemma 5. *Suppose that for all non-empty subsets N' of N, every unanimous and strategy-proof choice function φ' from $\mathbb{D}^{N'}$ to A is alternative decisive. Then, every unanimous and strategy-proof choice function φ from $\mathbb{D}^{N}$ to A is dictatorial.*

Proof. Let $N = \{1, 2, \dots, n\}$ and let N^k denote the set $\{1, 2, \dots, k\}$ for all $k = 1, \dots, n-1$. For all $k = 1, \dots, n-1$, define the choice function φ^k from $\mathbb{D}^{N^k}$ to A as follows: for all $p \in \mathbb{D}^{N^k}$, define $\varphi^k(p) = \varphi(p, R^{N \setminus N^k})$, where R is fixed linear order. Because φ is strategy-proof, it follows that φ^k is also strategy-proof. Note that if φ^k is dictatorial with dictator i_k in $\{1, 2, \dots, k\}$, then $\{i_k\}$ is decisive for φ at every profile q such that $q(j) = R$ for all $j \in N \setminus \{i_k\}$. Because φ is alternative decisive, this implies that φ is dictatorial with dictator i_k. So, dictatorship of φ^k implies dictatorship of φ.

Note that either N^{n-1} is decisive for φ or $\{n\}$ is decisive for φ. If $\{n\}$ is decisive for φ, then we have the desired result that φ is dictatorial. On the other hand, if N^{n-1} is decisive for φ, then φ^{n-1} must be unanimous, and hence it is alternative decisive. Again, note that either N^{n-2} or $\{n-1\}$ is decisive for φ^{n-1}. If $\{n-1\}$ is decisive for φ^{n-1}, then φ^{n-1} is dictatorial, and hence φ is dictatorial. On the other hand, if N^{n-2} is decisive for φ^{n-1}, as before, we proceed to φ^{n-2} and prove that either φ is dictatorial with dictator $n-2$ or φ^{n-3} is unanimous and strategy-proof.

Continuing in this manner, either we obtain that φ is dictatorial with dictator in $\{2, \dots, n\}$ or we conclude that $\{1\}$ is decisive at φ^1. However, if $\{1\}$ is decisive at φ^1, then φ is dictatorial with dictator 1. ■

A.1 Proof of Theorem 1

Proof. (*If part*) If part of the prove follows from Example 3.1.
(*Only-if part*) Let a domain $\mathbb{D}$ satisfy Condition 1. Suppose that $\mathbb{D}$ has no inseparable top-pair. By Condition 1, each alternative in $\tau(\mathbb{D})$ is top-connected to at least two different alternatives. As $\tau(\mathbb{D})$ is finite, this means we can number the elements in $\tau(\mathbb{D})$ as $x_0, x_1, x_2, \dots, x_{k-1}, x_k, x_{k+1}$ such that

(i) $x_0 = x_k$ and $x_1 = x_{k+1}$,

(ii) $\#\{x_{t-1}, x_t, x_{t+1}\} = 3$ for all $t \in \{1, 2, \dots, k\}$,

(iii) $x_t \leftrightsquigarrow x_{t+1}$ for all $t \in \{1, 2, \dots, k\}$, and

(iv) $\{x_1, x_2, \ldots, x_k\} = \tau(\mathbb{D})$.

Let φ from $\mathbb{D}^N$ to A be a unanimous and strategy-proof choice function. By Lemma 5, it is sufficient to show that φ is alternative decisive, which follows straightforwardly from Lemmas 1, 2, 3, and 4. ■

A.2 Proof of Theorem 2

Let a domain $\mathbb{D}$ satisfy Condition 1 and let $z^0, z^1, \ldots, z^k$ be a top-circuit in $\mathbb{D}$. Suppose that φ is a unanimous and strategy-proof choice function from $\mathbb{D}^N$ to A. It is sufficient to show that φ is not anonymous. Consider the domain $\widetilde{\mathbb{D}} = \cup_{t=1}^{k} \mathbb{D}^{z^t}$ and let $\widetilde{\varphi}$ is the restriction of φ to $\widetilde{\mathbb{D}}^N$. It is sufficient to prove that $\widetilde{\varphi}$ is dictatorial. Since φ is unanimous and strategy-proof, $\widetilde{\varphi}$ is also unanimous and strategy-proof. Let $\tau(\widetilde{\mathbb{D}}) = \{z^0, z^1, \ldots, z^k\}$ be the set of best alternatives in $\widetilde{\mathbb{D}}$. Then, the domain $\widetilde{\mathbb{D}}$ satisfies Condition 1, where the role of $\tau(\mathbb{D})$ is played by $\tau(\widetilde{\mathbb{D}})$. Note that the number of alternatives in $\tau(\widetilde{\mathbb{D}})$ is larger than or equal to 3. Therefore, by means of the fact that $\widetilde{\mathbb{D}}$ has no inseparable top-pair, Theorem 1 implies that $\widetilde{\varphi}$ is dictatorial.

Appendix B

For a subset C of A, let $\mathcal{M}(C) = \{a \in \mathcal{E} : ||a - c|| = ||a - d|| \text{ for all } c \text{ and } d \text{ in } C\}$. In words, $\mathcal{M}(C)$ consists of those points in space that are at an equal distance from all points in C. Then, $u \in \mathcal{M}(C_t)$ for all $1 \leq t \leq k$, where C_t is as defined in Section 4.1.

To show that $\mathbb{D}$ satisfies Condition 1, we introduce some basic geometrical notions. For two points a and b in the Euclidean space, let $[a, b] = \{c \in \mathcal{E} : ||a - c|| + ||c - b|| = ||a - b||\}$ be the closed line segment between a and b. Further, let $(a, b]$, $[a, b)$, and (a, b) have the related obvious meaning of half-open or open line segments between a and b. The perpendicular bisector of a and b, i.e., $\mathcal{M}(\{a, b\}) = \{c \in \mathcal{E} : ||a - c|| = ||c - b||\}$, divides $\mathcal{E}$ into two convex closed half spaces $\mathcal{H}_{ab} = \{c \in \mathcal{E} : ||a - c|| \leq ||c - b||\}$ and $\mathcal{H}_{ba} = \{c \in \mathcal{E} : ||b - c|| \leq ||c - a||\}$. Let $\mathcal{H}^0_{ab} = \{c \in \mathcal{E} : ||a - c|| < ||c - b||\}$ denote the open interior of $\mathcal{H}_{ab}$ and let $\mathcal{H}^0_{ba}$ denote that of $\mathcal{H}_{ba}$. Let E be a weak order on A. Define $\mathcal{V}(E) = \{u \in \mathit{convexhull}(A) : E_u = E\}$ as the set of bliss points with Euclidean order E. So, $\mathcal{V}(E) = \cap \{\mathcal{H}_{ab} : ab \in E \text{ and } ba \notin E\} \cap \mathit{convexhull}(A)$. Clearly, $\mathcal{V}(E)$ is a convex set. In the following remark, we present some basic geometric consequences.

REMARK B.1 (*Basics on indifference classes*). Clearly, we have (i) $\mathcal{M}(\{a, b\}) = \mathcal{H}_{ab} \cap \mathcal{H}_{ba}$, and (ii) $\mathcal{M}(C) = \cap\{\mathcal{M}(\{a, b\}) : a, b \in C\}$. This means if u and v are different points in $\mathcal{M}(\{a, b\})$, then the line through u and v, defined as $\mathit{line}(u, v) = \{u + \lambda(u - v) : \lambda \text{ being a real number}\}$, is contained in $\mathcal{M}(\{a, b\})$. By (ii), this means the $\mathit{line}(u, v)$ is contained in $\mathcal{M}(C)$ whenever u and v are different points in $\mathcal{M}(C)$.

Furthermore, for three different points a, b, and c,

$$\begin{aligned}\mathcal{M}(\{a,b,c\}) &= \mathcal{M}(\{a,b\}) \cap \mathcal{M}(\{a,c\}) \\ &= \mathcal{M}(\{a,b\}) \cap \mathcal{M}(\{b,c\}) \\ &= \mathcal{M}(\{b,c\}) \cap \mathcal{M}(\{a,c\}).\end{aligned}$$

This obviously generalizes to

$$\mathcal{M}(\{a_1, a_2, \ldots, a_k, x\}) = \mathcal{M}(\{a_1, x\}) \cap \mathcal{M}(\{a_2, x\}) \cap \ldots \cap \mathcal{M}(\{a_k, x\}).$$

B.1 Proof of Claim 1

Let $\mathbb{D}$ be a single-peaked Euclidean domain. (*Top-connectedness*) Let a and b be two locations in A. In order to prove that $\mathbb{D}$ is top-connected, it is sufficient to construct a top-connecting path from a to b. For real numbers λ between 0 and 1, consider preferences with bliss point $u_\lambda = (1-\lambda)a + \lambda b$. If $\lambda = 0$, then a is the most preferred alternative among A at any linear extension of E_{u_0}. Similarly, if $\lambda = 1$, then b is the most preferred alternative among A of any linear extension of E_{u_1}. Letting λ increase gradually from 0 to 1, we obtain a continuous path of Euclidean preferences E_{u_λ}. In what follows, we argue that one can find a top-connecting path from a to b among the extensions of these preferences.

As A is finite, $\mathbb{D}$ is also finite. This, together with the fact that $\mathcal{V}(E) \cap [a,b]$ is convex for every Euclidean order E, implies there is an integer l and real numbers $\lambda_0 = 0 < \lambda_1 < \cdots < \lambda_l = 1$ such that $E_{u_\alpha} = E_{u_\beta}$ for all $t \in \{1, \ldots, l\}$ and all $\alpha, \beta \in (\lambda_{t-1}, \lambda_t)$. Let $E_{p_\alpha} = C_1^\alpha C_2^\alpha \ldots C_{k^\alpha}^\alpha$ and $E_{p_\beta} = C_1^\beta C_2^\beta \ldots C_{k^\beta}^\beta$ for all $\alpha \in (\lambda_{t-1}, \lambda_t)$ and all $\beta \in (\lambda_t, \lambda_{t+1})$. If C_s^α is not singleton for some $s \in \{1, \ldots, k\}$, then by Remark B.1, we have that the line segment $[a,b] \subseteq \mathcal{M}(C_s)$. So, for all i in $\{1, \ldots, k^\alpha\}$, there is a unique j in $\{1, \ldots, k^\beta\}$ such that $C_i^\alpha \subseteq C_j^\beta$. Similarly, for all i in $\{1, \ldots, k^\beta\}$, there is a unique j in $\{1, \ldots, k^\alpha\}$ such that $C_i^\beta \subseteq C_j^\alpha$. Hence, for all i in $\{1, \ldots, k^\alpha\}$, there is a unique j in $\{1, \ldots, k^\beta\}$ such that $C_i^\alpha = C_j^\beta$. Consequently, we have $k^\alpha = k^\beta$. Without loss of generality, let us assume $C_1^\alpha = C_1^\beta$, $C_2^\alpha = C_2^\beta, \ldots, C_{i-1}^\alpha = C_{i-1}^\beta$ and $C_i^\alpha = C_{i+s}^\beta$. Note that in order to have such a top-connected path from a to b, it is sufficient to show that $u_{\lambda_t} \in \mathcal{M}(C_{i+s}^\alpha \cup C_{i+s-1}^\alpha \cup \ldots \cup C_{i+1}^\alpha \cup C_i^\alpha)$.

Without loss of generality, assume $i = 1$ and that $C_1^\beta = C_s^\alpha$, where $s > 1$. We have that the line segment $(u_{\lambda_0}, u_{\lambda_1})$ is contained in $\mathcal{H}_{xy}$ and the line segment $(u_{\lambda_1}, u_{\lambda_2})$ is contained in $\mathcal{H}_{yx}$ for all (x,y) in $(C_1^\alpha \cup C_2^\alpha \cup \ldots \cup C_{s-1}^\alpha) \times C_s^\alpha$ and (y,x) in $(C_1^\beta \cup C_2^\beta \cup \ldots \cup C_{t-1}^\beta) \times C_t^\beta$. Therefore, u_{λ_t} is on the boundary of both $\mathcal{H}_{xy}$ and $\mathcal{H}_{yx}$, and consequently in $\mathcal{H}_{xy} \cap \mathcal{H}_{yx} = \mathcal{M}(\{x,y\})$. As this holds for all (x,y) in $(C_1^\alpha \cup C_2^\alpha \cup \ldots \cup C_{s-1}^\alpha) \times C_s^\alpha$, by Remark B.1, it follows that $u_{\lambda_t} \in \mathcal{M}(C_1^\alpha \cup C_2^\alpha \cup \ldots \cup C_s^\alpha)$.

(*Pervasiveness*) Let u be a point in *convexhull*(A) and let R be a linear extension of E_u such that $R \equiv ab \cdots$

for some a and b are in A. It is sufficient to prove that there are linear orders R' in $\mathbb{D}$ with $R' \equiv ba\cdots$. Take any alternative $c \in A \setminus \{a,b\}$. Consider the line segment $[u,b]$. Clearly, as $R \equiv ab\cdots c\cdots$, we have that $u \in \mathcal{H}_{bc}$. By definition, $b \in \mathcal{H}^0_{bc}$. So, the line segment $(u,b]$ is contained in $\mathcal{H}^0_{bc}$. Let d be the intersection of $\mathcal{M}(\{a,b\})$ and the line segment $(u,b]$. As before, by considering bliss points u_λ along this line segment, we obtain a top-connected path from a to b such that on the line segment $(u,d]$, we can choose extensions R'' where a is ordered above b. Now, as c is chosen arbitrarily, we have that $R'' \equiv ab\cdots$. As d is in $\mathcal{M}(\{a,b\})$, we can choose extension $R' \equiv ba\cdots$ of E_d. This proves that the top-connections are pervasive.

(*Richness*) Considering the top-connecting paths constructed above, we can choose top-connecting preferences as strongly top-connected. By Remark 2.1, it follows that the domain $\mathbb{D}$ satisfies the richness property of Condition 1.

B.2 Proof of Corollary 2

Theorem 2 proves the *only-if* part. For the *if part*, let $\mathbb{D}$ have no top-circuits. So, the graph $\mathcal{G}$ is a tree over A. In view of Demange (1982), it is sufficient to show that $\mathbb{D}$ is single-peaked on $\mathcal{G}$, i.e., if an alternative b is on a path from some alternative a to some other alternative c, then in any order R in $\mathbb{D}$, b cannot be the worst alternative amongst a, b, and c, that is, we will have either $bc \in R$ or $ba \in R$. Assume for contradiction that for some $u \in \text{convexhull}(A)$, there is an extension R of E_u such that $cb \in R$ and $ab \in R$. Note that a, c and u are in $\mathcal{H}_{ab} \cap \mathcal{H}_{cb}$. So, we can construct a top-connecting path from a to c by taking bliss points first along the line segment $[a,u]$ and then along the line segment $[u,c]$. As all the bliss points v on these line segments are in $\mathcal{H}_{ab} \cap \mathcal{H}_{cb}$, we can chose extensions R' of E_v such that $ab \in R'$ and $cb \in R'$. This results in a top-connected path from a to c that does not go via b. This contradicts the fact that $\mathcal{G}$ is a tree.

B.3 Proof of Claim 2

The proof of Claim 2 follows by using similar arguments as for the proof of Claim 1.

B.4 Proof of Claim 3

The *if part* of the proof follows from the definition. For the *only-if part*, suppose ab is an inseparable top-pair of $\mathbb{D}$. It is sufficient to prove that $\tau(\mathbb{D}) = \{a,b\}$. Assume for contradiction that $R \equiv c\cdots$ for some $c \in \tau(\mathbb{D}) \setminus \{a,b\}$ and some $R \in \mathbb{D}$. We distinguish the following two cases.

Case 1. Suppose $R \equiv c\cdots a\cdots b\cdots$. Consider $R^{ab} \equiv ab\cdots$. Let u be a point in A such that R is an extension of $-E_u$, i.e., $R \subseteq -E_u$. Let v be a point in A such that R^{ab} is an extension of $-E_v$. Consider the line segment $[u,v]$ of dips and a path of top-connections in $\mathbb{D}$ constituted with the extensions R_r of $-E_r$ for all $r \in [u,v]$. As u and v are both in $\mathcal{H}_{ba}$, we can choose these extensions such that $ab \in R_r$. However, as a is the best alternative in R_v, at some point on this path of top-connections, a must be directly top-connected to some alternative in $\tau(\mathbb{D}) \setminus \{b\}$. This violates the inseparability of ab.

Case 2. Suppose $R \equiv c \cdots b \cdots a \cdots$. Consider $R^{ab} \equiv ab \cdots$. Let w be a point in A such that R is an extension of $-E_w$. As A is the convex hull of a finite number of points, we can assume that $w \notin \mathcal{M}(\{b,c\})$. Let v be a point in A such that R^{ab} is an extension of $-E_v$. Consider the line segment $[w,v]$ of dips. Note that w is in $\mathcal{H}_{ba}$ and v is in $\mathcal{H}_{ab}$. So, the line segment $[w,v]$ intersects $\mathcal{M}(\{a,b\})$, say at x. If $\mathcal{M}(\{b,c\})$ intersects the line segment $[w,x)$, say at y, then $\cdots b \cdots c \cdots a \cdots \equiv R_z$ for all extensions R_z of $-E_z$, where z is on the line segment (y,x). This means $\cdots a \cdots b \cdots \equiv R_z$ and $\cdots b \cdots c \cdots \equiv R_z$ for all z in the line segment $(x,v]$, and hence $x \in \mathcal{M}(\{a,b\}) \cap \mathcal{M}(\{a,c\})$. However, $\mathcal{M}(\{a,b\}) \cap \mathcal{M}(\{a,c\}) \subseteq \mathcal{M}(\{b,c\})$, which means $[w,v] \subseteq \mathcal{M}(\{b,c\})$. This cannot be true by our choice of w. So, we may assume that both w and x are in $\mathcal{H}_{bc}$, and therewith $\cdots c \cdots b \cdots a \cdots \equiv R_z$ for all R_z extending $-E_z$, where z lies on the line segment $[w,x)$. This means there is $d \in \tau(\mathbb{D}) \setminus \{a,b\}$ such that $d \cdots a \cdots b \cdots \equiv R_x$, where R_x extends $-E_x$. However, Case 1 shows that this is not possible. This completes the proof for this case.

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